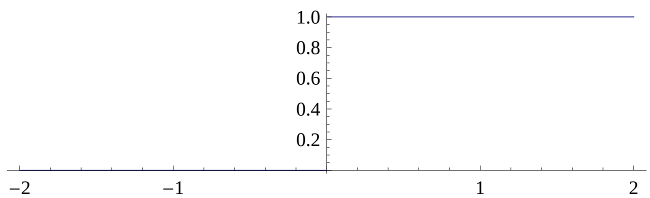
Step Function Algebra

How algebraic functions of step functions of algebraic functions combine

The basic Heaviside step function u[t]

```
Plot[UnitStep[t], {t, -2, 2}, AspectRatio → Automatic]
```



Step functions of functions of the independent variable t

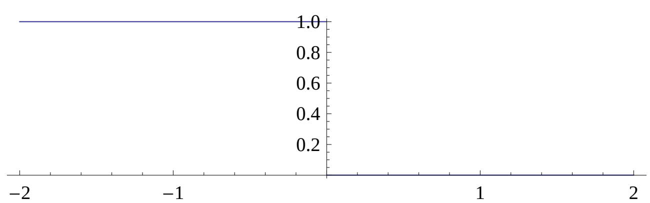
Arithmetic functions of the independent variable are the simplest.

- **General Rules for linear arithmetic functions of the independent variable u[a*t+b]**

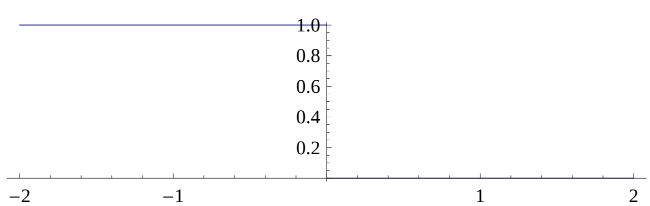
Rule 1: Multiplication by a positive value has no effect but multiplication by a negative value causes reflection in vertical axis (the step function is odd) but no change in height

- **a < 0**

```
a = -1;
b = 0;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```

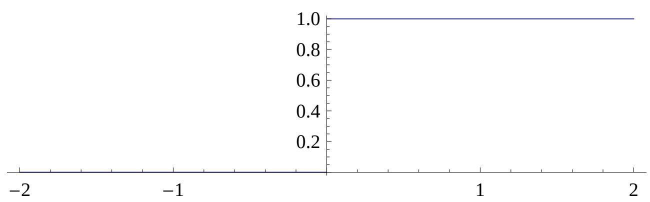



```
a = -2;
b = 0;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```

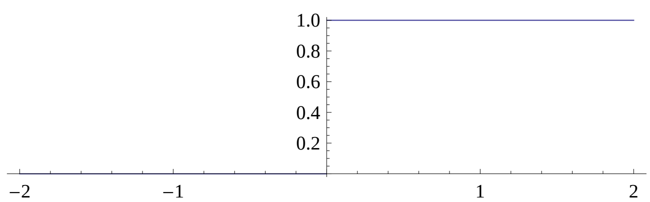


- **a > 0**

```
a = +1;
b = 0;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```



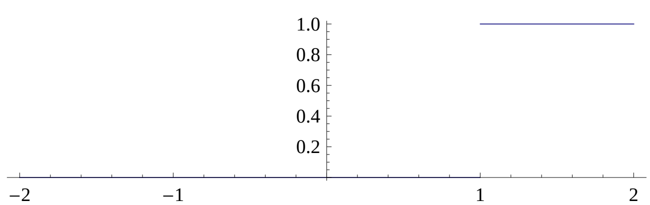

```
a = +2;
b = 0;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```



Rule 2: Addition causes a shift along the t-axis

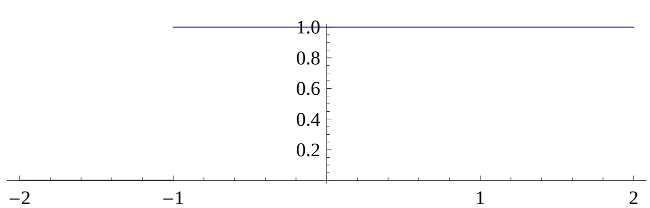
- **b < 0**

```
a = +1;
b = -1;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```



- **b > 0**

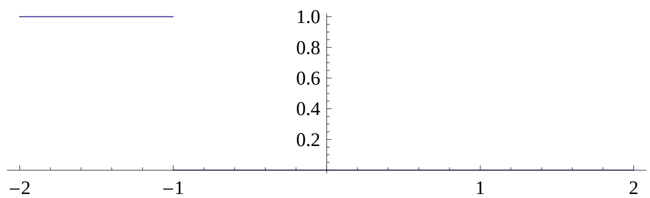
```
a = +1;
b = +1;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```



- **Combination of shift and reflection (these operations do not commute - do shift before reflection)**

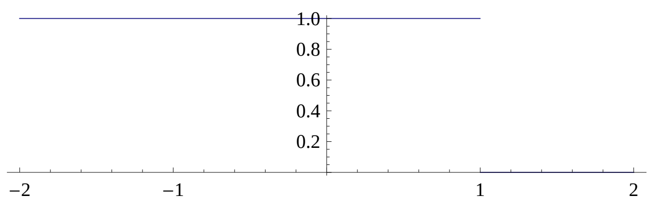
- **a<0, b < 0**

```
a = -1;
b = -1;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```



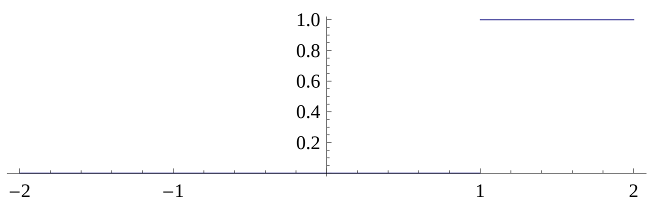
- **a<0, b >0**

```
a = -1;
b = +1;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```



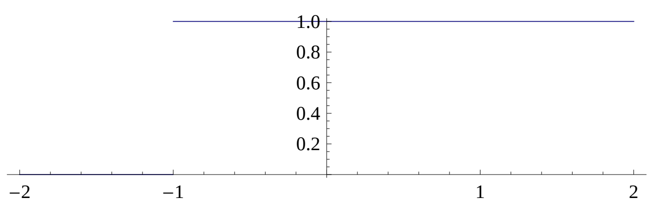
- **a>0, b <0**

```
a = +1;
b = -1;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```



- **a>0, b >0**

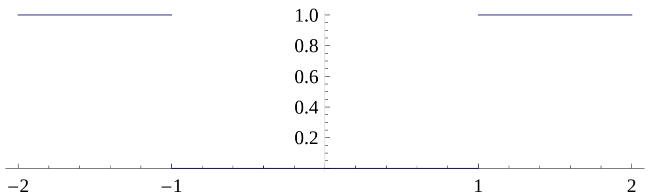
```
a = +1;
b = +1;
Plot[UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```



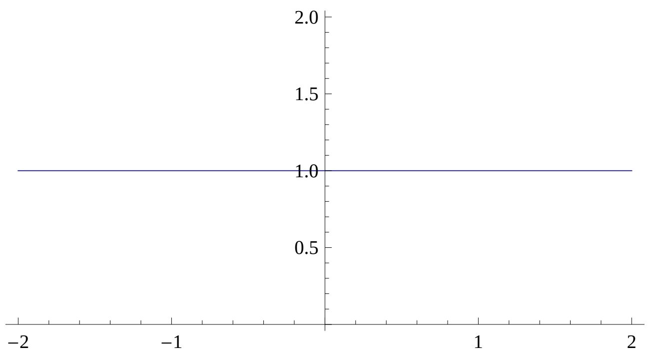
Nonlinear arithmetic functions of the independent variable u[f(t)]

- **Nonlinear polynomial functions**

```
a = +1;
b = -1;
Plot[UnitStep[a t^2 + b], {t, -2, 2}, AspectRatio → Automatic]
```

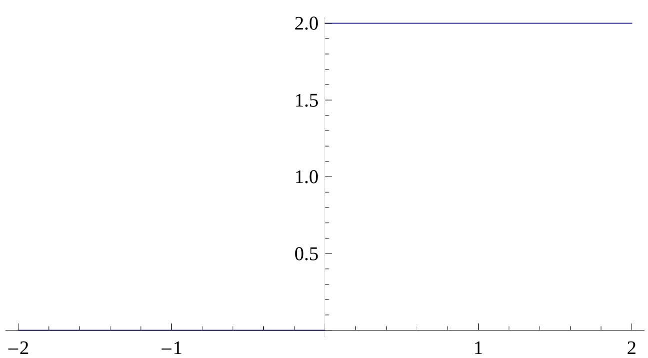



```
a = +1;
b = +1;
Plot[UnitStep[a t^2 + b], {t, -2, 2}, AspectRatio → Automatic]
```

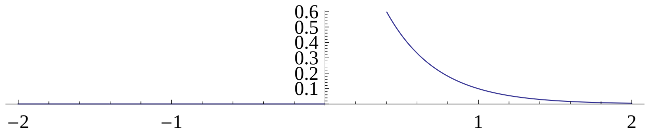


- **Multiplication of step function by an algebraic function**

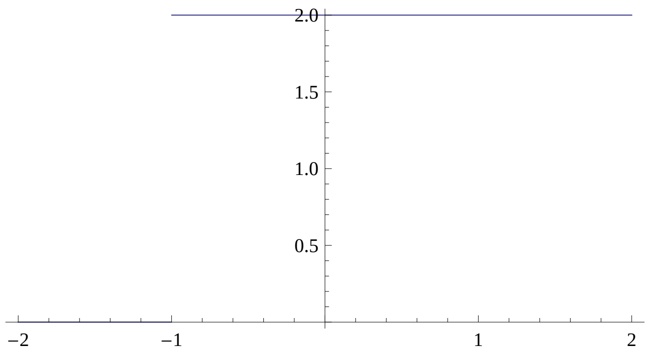
```
a = +1;
b = 0;
c = +2;
Plot[c UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```



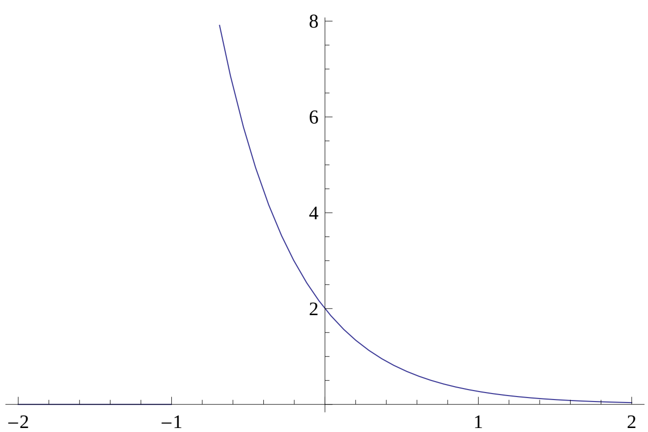
```
a = +1;
b = 0;
c = +2;
Plot[c ⅇ^(-3 t) UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```




```
a = +1;
b = +1;
c = +2;
Plot[c UnitStep[a t + b], {t, -2, 2}, AspectRatio → Automatic]
```




```
a = +1;
b = +1;
c = +2;
Plot[c ⅇ^(-2 t) UnitStep[a t + b], {t, -2, 2}]
```



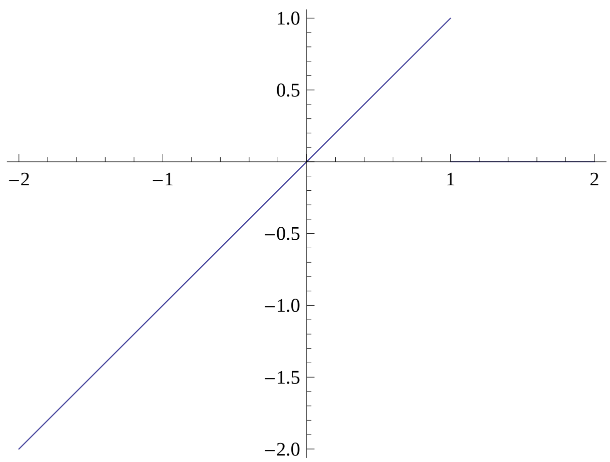
Combination of step functions

Total behavior is a combination of separate component behaviors

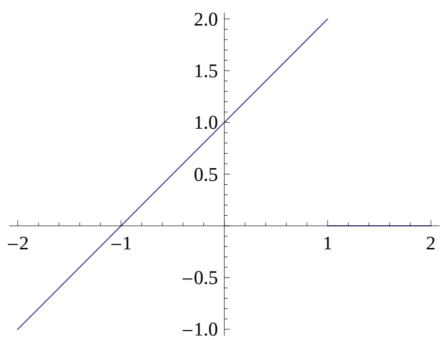
- **Arithmetic combination of step expressions: addition and multiplication**

- **First, let's multiply a unit functions by some algebraic expression**

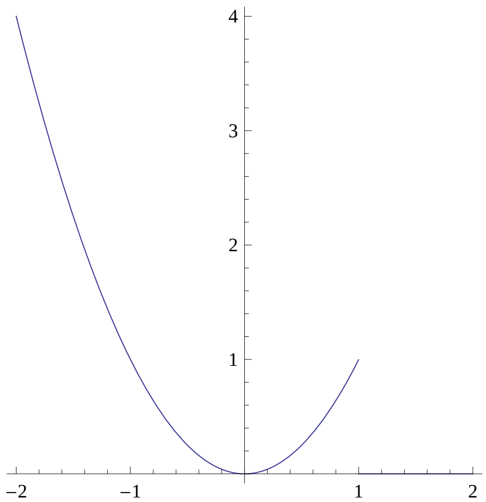
```
Plot[t UnitStep[-t + 1], {t, -2, 2}, AspectRatio → Automatic]
```



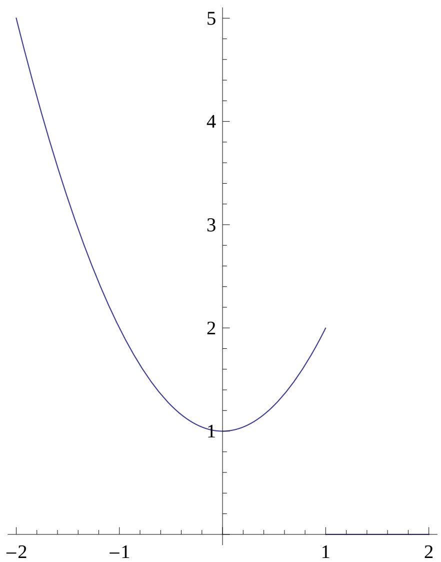
```
Plot[(t + 1) UnitStep[-t + 1], {t, -2, 2}, AspectRatio → Automatic]
```



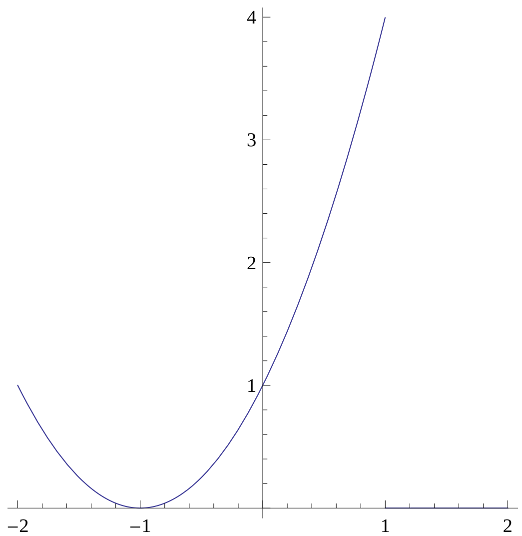
```
Plot[t^2 UnitStep[-t + 1], {t, -2, 2}, AspectRatio → Automatic]
```




```
Plot[(t^2 + 1) UnitStep[-t + 1], {t, -2, 2}, AspectRatio → Automatic]
```

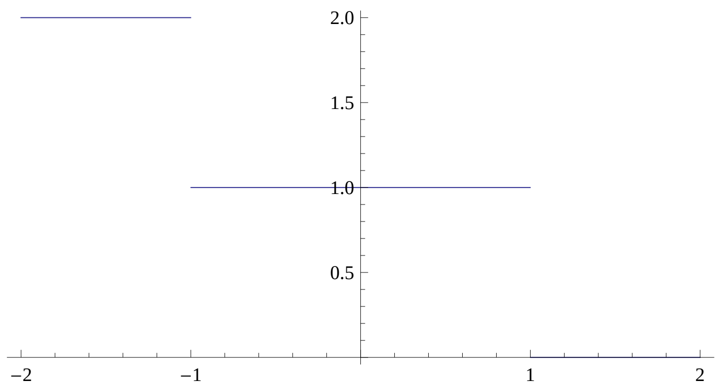


```
Plot[(t + 1)^2 UnitStep[-t + 1], {t, -2, 2}, AspectRatio → Automatic]
```

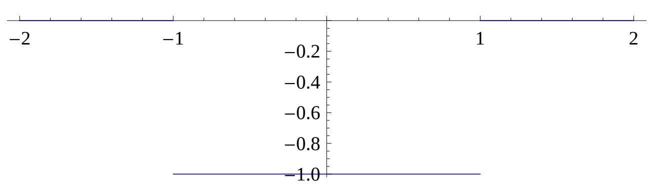


- **Next, let's add and subtract two step functions**

```
Plot[UnitStep[-t - 1] + UnitStep[-t + 1], {t, -2, 2}, AspectRatio → Automatic]
```

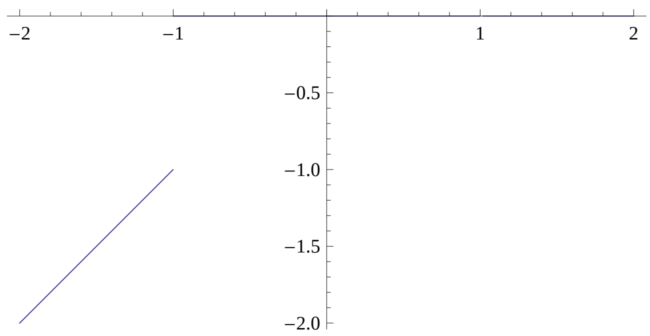



```
Plot[UnitStep[-t - 1] - UnitStep[-t + 1], {t, -2, 2}, AspectRatio → Automatic]
```



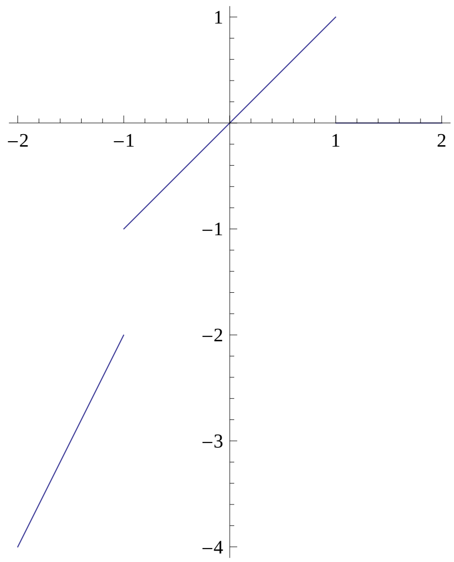
- **Now, let's multiply two step functions**

```
Plot[t (UnitStep[-t - 1] UnitStep[-t + 1]), {t, -2, 2}, AspectRatio → Automatic]
```



- **Finally, let's multiply the result of arithmetic combination of step functions by an algebraic function**

```
Plot[t (UnitStep[-t - 1] + UnitStep[-t + 1]), {t, -2, 2}, AspectRatio → Automatic]
```




```
Plot[t (UnitStep[-t - 1] - UnitStep[-t + 1]), {t, -2, 2}, AspectRatio → Automatic]
```