


Chapter 6

CONFIDENCE INTERVALS

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 Using Excel 2007 to obtain the 95% Confidence Interval	

WHAT IS A CONFIDENCE INTERVAL?

- ① A confidence interval is a range of scores believed with some level of confidence to contain the true population value
- ② From a random sample, we obtain a sample statistic (e.g., the sample mean, the sample correlation) that is assumed to be an estimate of the population parameter (e.g., population mean, population correlation)
- ③ The amount the sample statistic is different from the population parameter is referred to as sampling error
 - Sampling error is caused by random error and sample size

SAMPLING ERROR:

- ① The larger the size of the sample, the more accurate the sample will estimate the population value
 - The larger the sample size, the smaller the sampling error
- ② Imagine randomly sampling 25 people from a population, measuring their heights (in inches), and obtaining the sample mean and sample SD
 - Then put the 25 people back into the population
- ③ Now imagine repeating this process for an infinite number of samples, each containing 25 people

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All possible sample means, each of which is based on a sample size of 25 people.

$$\bar{X}_1 = 68.5$$

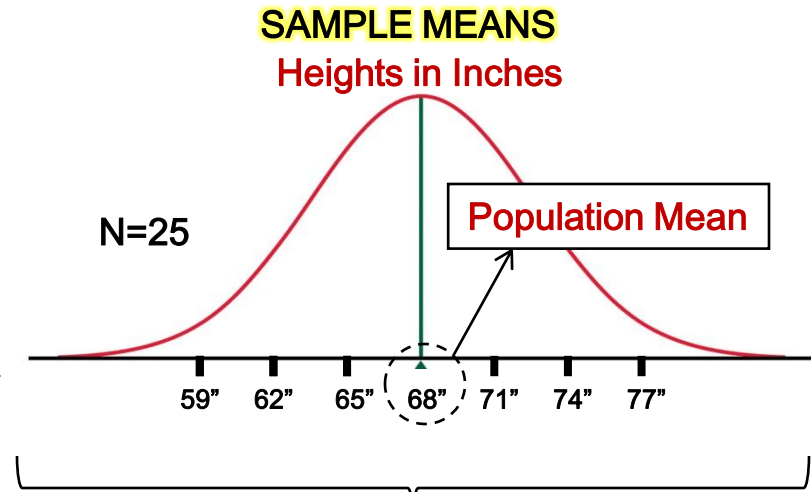
$$\bar{X}_2 = 69.25$$

$$\bar{X}_3 = 67.75$$

$$\bar{X}_4 = 67.5$$

$$\bar{X}_5 = 68.75$$

$$\bar{X}_\infty =$$



Sampling distribution of samples means, where the mean is equal to the population mean and the standard deviation is equal to the standard error of the mean (SEM).

SAMPLING ERROR:

- 1 As the sample size increases:
 - the sample means will become more accurate in estimating the population mean
 - the sample means will become more similar to each other
 - the estimated SEM will become smaller, and
 - the shape of the sampling distribution (i.e., histogram) will become narrower and taller

SAMPLING ERROR AND CONFIDENCE INTERVALS:

- 1 A sample mean is an estimate of the population mean plus or minus sampling error

Sample Mean \pm Sampling Error

- The SEM is an estimate of the sampling error
- 2 The size of the sampling error can be reduced by increasing the sample size
 - 3 As sample size increases, the sample mean becomes a more accurate estimate of the population mean

SAMPLING ERROR AND CONFIDENCE INTERVALS:

- ① A confidence interval builds a range of values around a sample mean
- ② The size of a confidence interval is a function of:
 - Sample size (larger $N \rightarrow$ smaller sampling error) and
 - The confidence level chosen
- ③ Typically, researchers choose a confidence level of 90%, 95%, or 99%
- ④ Thus, the confidence interval is:

Sample Mean \pm Sampling Error(Level of Confidence)

CONFIDENCE INTERVALS:

- 1 The formula for the 95% confidence interval is:

$$\text{Sample Mean} \pm \underbrace{\text{SEM}(t_{.05})}_{\text{Margin of Error}}$$

Where:

- the SEM is an estimate of the sampling error and
- $t_{.05}$ is the value from the t -distribution, where 5% of all t -scores are expected to be equal to or less than this score

STUDENT'S t DISTRIBUTION

- ① The t distribution is a z distribution that has been adjusted for:
 - Sample size and
 - Not knowing the population SD (we have to estimate it using the sample SD)
- ② t - scores:
 - Are interpreted similarly to z scores (SD units), but are estimated SD units
 - Have a mean equal to zero (just like z scores)
 - Equal z – scores when sample size equals ∞

STUDENT'S t DISTRIBUTION

- 1 The t distribution is located on the class website under the statistical tables link
- 2 The value of $t_{.05}$ is found by:
 - Determining the degrees of freedom (df)
 - Formula: $df = N - 1$
 - Locate the $t_{.05}$ value under the 95% Confidence Interval heading
- 3 What is the value for $t_{.05}$ when $N = 25$?

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STUDENT'S t DISTRIBUTION

When $N = 25$, the value of $t_{.05} = \pm 2.06$

$t_{.05}$

$N = 25$

TABLE B t Distribution Critical Values

df	Confidence Level					
	80%	90%	95%	98%	99%	99.8%
	Right-Tail Probability					
	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.611
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450

EXAMPLE 1:

- ① Suppose a sample of 16 light trucks is randomly selected off the assembly line. The trucks are driven 1000 miles and the fuel mileage (MPG) of each truck is recorded. It is found that the mean MPG is 22 with a SD equal to 3.
- What is the 95% confidence interval?

➔ **Sample Mean \pm SEM($t_{.05}$)**

➔ $22 \pm \frac{3}{\sqrt{16}}$ **0.131**

\swarrow $df = 16 - 1 = 15$

EXAMPLE 1:

$$\rightarrow 22 \pm .75 \left(\frac{.131}{\sqrt{16}} \right)$$

$$\rightarrow 22 \pm 1.598$$

95% Confidence Interval:

Lower Limit $\rightarrow 20.402$

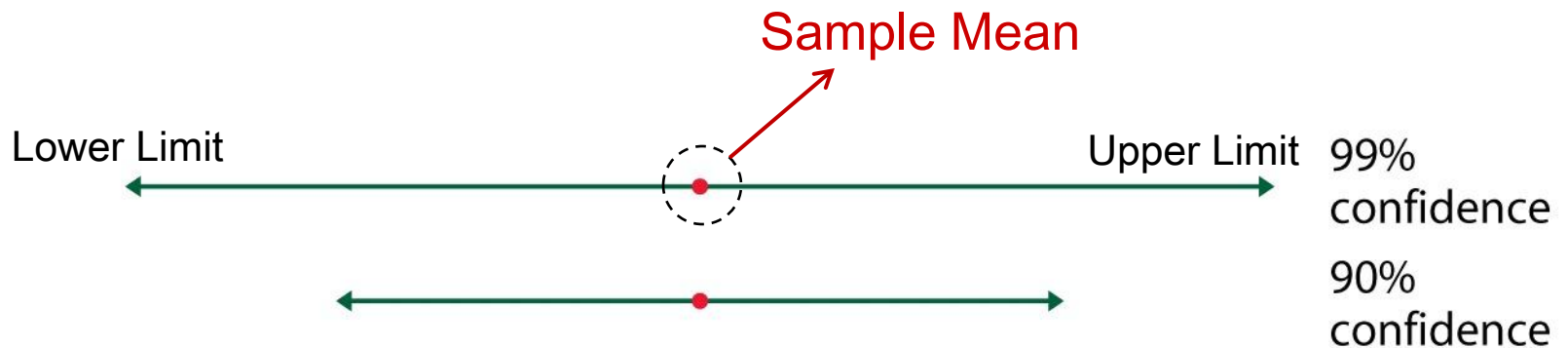
Upper Limit $\rightarrow 23.598$

① Interpretation:

- We are 95% confident that the true fuel efficiency for all light trucks manufactured at the plant to be in the range of 20.402 MPG to 23.598 MPG. (N = 16)

CONFIDENCE LEVELS

- 1 The confidence level chosen (e.g., 99% versus 95%) affects the size (or range) of the confidence interval



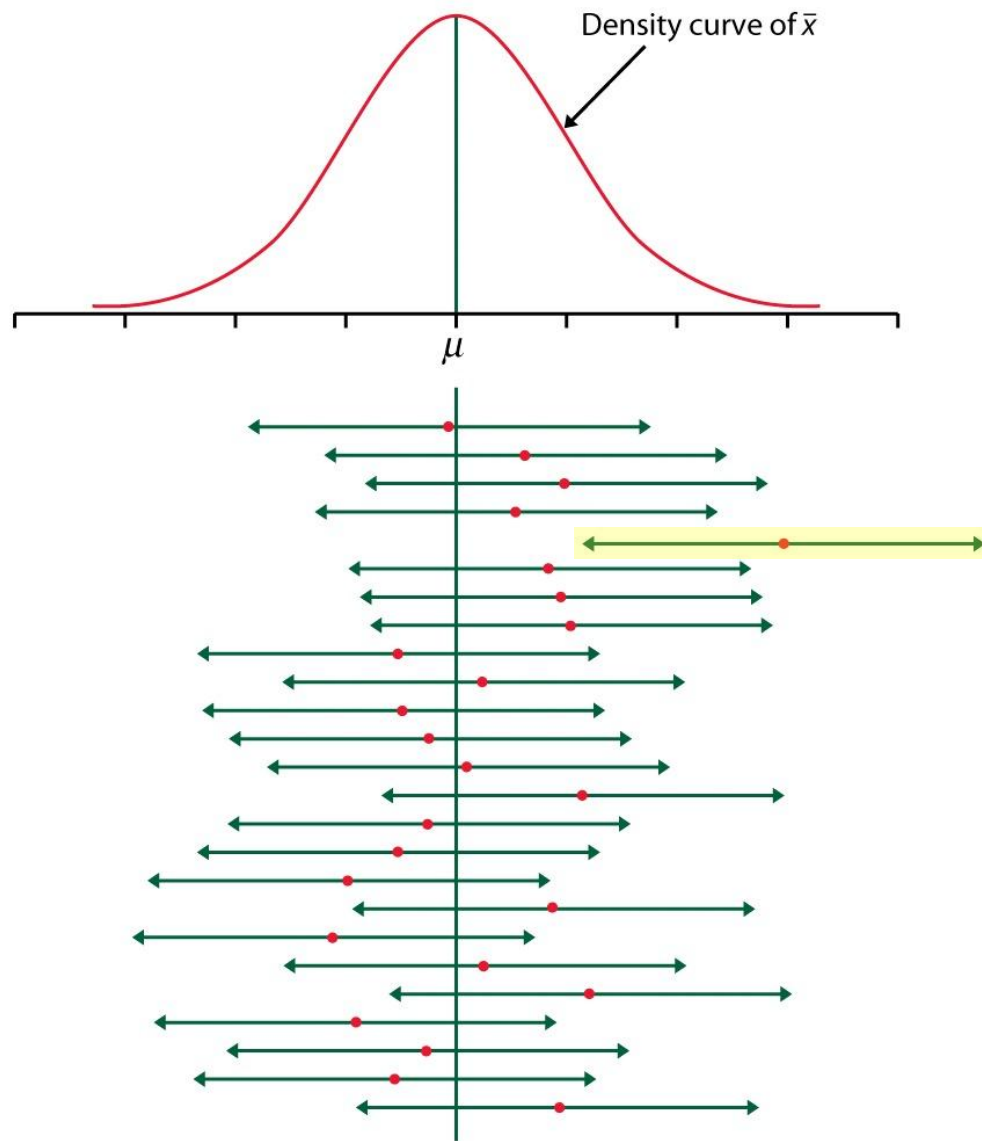
- 2 The higher the confidence level chosen, the wider the confidence interval
 - Because we want to be more confident (99% vs. 95%), the interval must be larger

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CONFIDENCE LEVELS

- 1 The level of confidence chosen tells us what to expect in the *long run*
- 2 A 95% confidence level means that we expect 5 out of 100 sample means to give us intervals that DO NOT contain the true population mean



EXAMPLE 2:

- ① Suppose you want to rent an unfurnished one-bedroom apartment for next semester. The mean monthly rent for a random sample of 10 apartments advertised in the local paper is \$540 .with a sample SD equal to \$80.
- What is the 95% confidence interval?

➔ Sample Mean \pm SEM($t_{.05}$)

➔ $540 \pm \frac{80}{\sqrt{10}} \cdot 2.262$

$df = 16 - 1 = 15$

EXAMPLE 2:

$$\rightarrow 540 \pm 25.298 \cdot 0.262$$

$$\rightarrow 540 \pm 57.225$$

95% Confidence Interval:

Lower Limit \rightarrow \$482.78

Upper Limit \rightarrow \$597.23

① Interpretation:

- We are 95% confident that the true fuel efficiency for all light trucks manufactured at the plant to be in the range of \$482.78 to \$597.23. (N = 10)

EXAMPLE 2:

- ① Using the same example, suppose we sample 20 adds instead of 10. What affect will this have on the confidence interval?
- ANSWER → The confidence interval will be smaller because the sample size is larger. A larger sample size will reduce the size of the sampling error (or SEM)

$$\rightarrow 540 \pm \frac{80}{\sqrt{20}} \cdot 2.093$$

df = 20 - 1 = 19

EXAMPLE 2:

$$\begin{aligned} &\rightarrow 540 \pm 17.889 \cdot 0.093 \\ &\rightarrow 540 \pm 37.441 \end{aligned}$$

95% Confidence Interval:

Lower Limit \rightarrow \$502.56

Upper Limit \rightarrow \$577.44

① Interpretation:

- We are 95% confident that the true monthly mean rent for unfurnished one-bedroom apartments in the community is in the range of \$502.56 to \$577.44. (N = 20)

EXAMPLE 2:

95% Confidence Interval (N = 10):

Lower Limit → \$482.78

Upper Limit → \$597.23

95% Confidence Interval (N = 20):

Lower Limit → \$502.56

Upper Limit → \$577.44

- 1 Increasing the sample size from $N = 10$ to $N = 20$ reduced the size of the confidence interval by nearly \$20 on each side of the interval

EXAMPLE 2:

- ① Using the same example where $N = 10$, suppose we choose a confidence level of 99% instead of 95%. What affect will this have on the size of the confidence interval?
- ANSWER → The confidence interval will be larger because we want to be even more certain that the interval contains the true population mean

$$\rightarrow 540 \pm \frac{80}{\sqrt{10}} \cdot 2.821$$

*t*_{.05} under 99% column in *t* table

df = 10 - 1 = 9

EXAMPLE 2:

$$\begin{aligned} &\rightarrow 540 \pm 25.298 \left(\frac{0.821}{\sqrt{10}} \right) \\ &\rightarrow 540 \pm 71.366 \end{aligned}$$

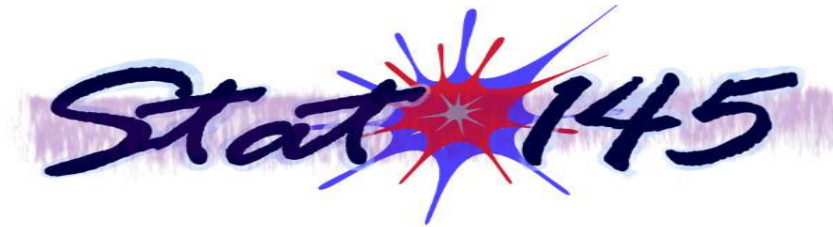
95% Confidence Interval:

Lower Limit \rightarrow \$468.63

Upper Limit \rightarrow \$611.37

① Interpretation:

- We are 99% confident that the true monthly mean rent for unfurnished one-bedroom apartments in the community is in the range of \$468.63 to \$611.37. (N = 10)



THAT'S IT FOR CHAPTER 7