Chapter 8

The One-Sample *t* - test

WHAT ARE THE FOUR STEPS IN AN EXPERIMENT?

1. State the hypotheses: What is the null hypothesis (Ho)? What is the alternative hypothesis(HA)?
2. Test the null hypothesis at the desired *p* value using the appropriate statistical test: Since this problem compares a single sample mean to a population mean where the population SD is known, we can use the one sample z-test
3. State your statistical conclusion regarding the null hypothesis (i.e., reject or fail to reject Ho)
4. Provide an interpretation of your statistical conclusion

WHEN IS THE ONE-SAMPLE *t* – TEST USED?

❶ To determine if the difference between a single sample mean and a known or estimated population mean (or μ) is statistically significant

❷ When the population SD (or σ) is unknown and

❸ When the data are of at least interval or ratio scales

$$t_{obt} = \frac{\overline{X} - \mu}{\frac{SD}{\sqrt{n}}}$$

- $\overline{X}$: Sample Mean
- μ: Population Mean
- SD: Sample Standard Deviation
- n: Sample Size
- $\frac{SD}{\sqrt{n}}$: Estimated Standard Error of the mean (SEM)

EXAMPLE 1: One-sample *t* – test

❶ Suppose the average annual rainfall for the local area was previously known to be 8 inches. A local meteorologist believes there was above average rainfall from 1997 thru 2001 and argues that the average annual rainfall during this period was significantly different from the overall average annual rainfall of 8 inches. The average annual rainfall recorded from 1997 thru 2001 are given below.

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	8	5	7	5	6

SD → 1.30 Sample Mean= 6.2 N = 5

WE'LL USE THE ONE-SAMPLE *t* - TEST:

❶ We're comparing a single sample mean (6.2 inches) to a known population mean (8 inches)

❷ We only know the sample SD (1.30 inches)

❸ The data are ratio scale

WHEN IS THE ONE-SAMPLE *t* – TEST USED?

❶ Null Hypothesis:

- The average annual rainfall from 1970 thru 2010 is the same as the overall average annual rainfall of 8 inches. Any observed difference is solely due to random error.

❷ Alternative Hypothesis:

- The average annual rainfall from 1970 thru 2010 was not the same as the overall average annual rainfall of 8 inches, but was significantly higher. The observed difference is not solely due to random error, but indicates a real difference in average annual rainfall.

FILLING IN THE FORMULA:

❶ Sample Mean = 6.2 inches SD = 1.30 inches N = 5

$$t_{obt} = \frac{6.2 - 8}{\frac{1.30}{\sqrt{5}}} = -3.10$$

HOW DO WE KNOW WHEN TO REJECT THE NULL HYPOTHESIS?

❶ **The null hypothesis is rejected when:**

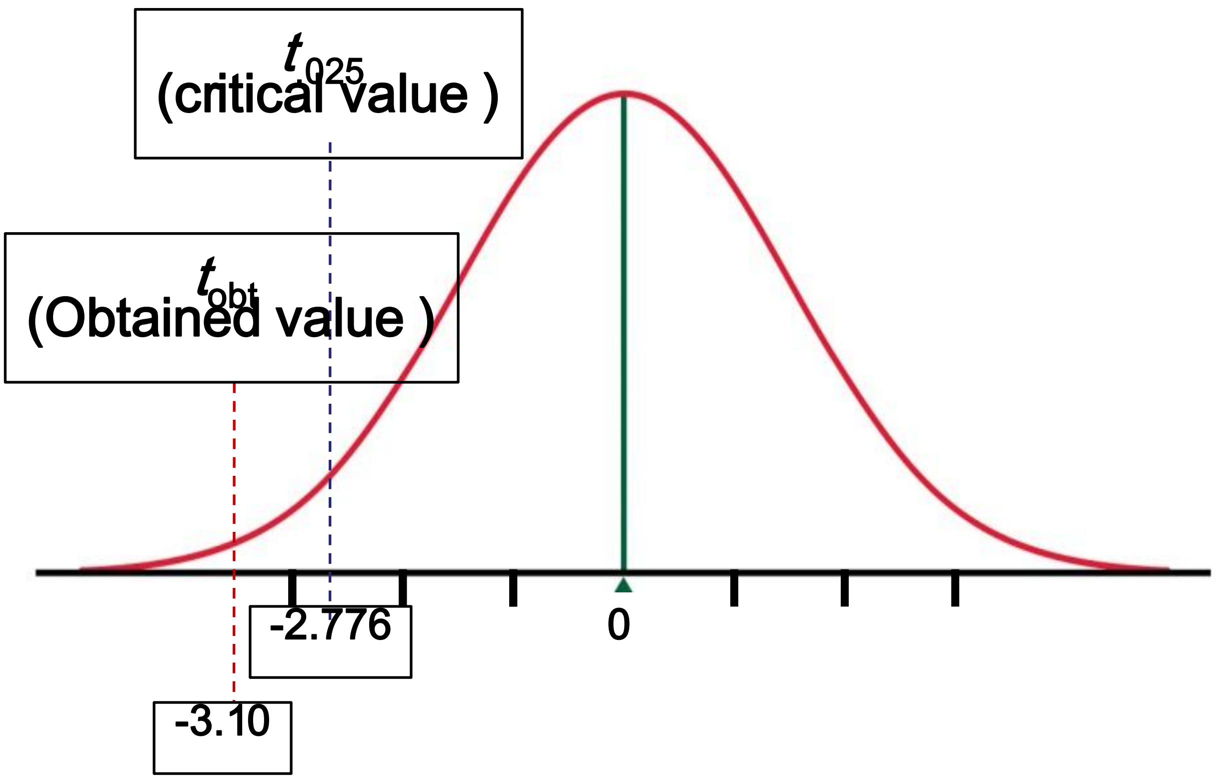
t_{obt} is equal to or more extreme than $t_{.025}$

- Where $t_{.025}$ is the critical value from the t distribution and is found using:

$$df = N - 1 \quad \text{OR} \quad 5 - 1 = 4$$

WHEN IS THE ONE-SAMPLE *t* – TEST USED?

t Distribution



❶ The null hypothesis is rejected since the obtained value is more extreme than the critical value ($p = .05$)

STATISTICAL CONCLUSION:

❶ "Since $t(4) = -3.10$, $p < .05$; Reject the null hypothesis."

- Where $p < .05$ means that if the null hypothesis is true, the probability in the *long-run,* of obtaining a t-value of -3.10 or more extreme due to random error alone is less than 5%."

- Remember, the critical value, $t_{.025}$, is the point where 5% or less of all scores are at or beyond (or more extreme)

INTERPRETATION:

❶ "It appears that there was less-than average rain in the local area from 1999 to 2001. The observed average rainfall for this period does not appear to be due to random error alone, but suggests that the weather pattern for the local area was different during the period studied."

EXAMPLE 2: One-sample *t* - test

1. Suppose a sample of 16 light trucks is randomly selected off the assembly line. The trucks are driven 1000 miles and the fuel mileage (MPG) of each truck is recorded. It is found that the mean MPG is 22 with a SD equal to 3. The previous model of the light truck got 20 MPG.

2. Questions:
   - State the null hypothesis for the problem above
   - Conduct a test of the null hypothesis at $p = .05$. BE SURE TO PROPERLY STATE YOUR STATISTICAL CONCLUSION.
   - Provide an interpretation of your statistical conclusion using the variables from the description given

EXAMPLE 2: Null Hypothesis:

- ❶ We expect the sample of 16 light trucks to get the same average MPG as the previous model. Any observed difference in MPG between the new light trucks and the previous model is assumed to be solely due to random error.

EXAMPLE 2: Test of the Null Hypothesis

❶ Sample Mean = 22MPG SD = 3 MPG N = 5

Population Mean = 20 MPG

$$t_{obt} = \frac{22 - 20}{\dfrac{3}{\sqrt{16}}} = 2.67$$

EXAMPLE 2: Test of the Null Hypothesis

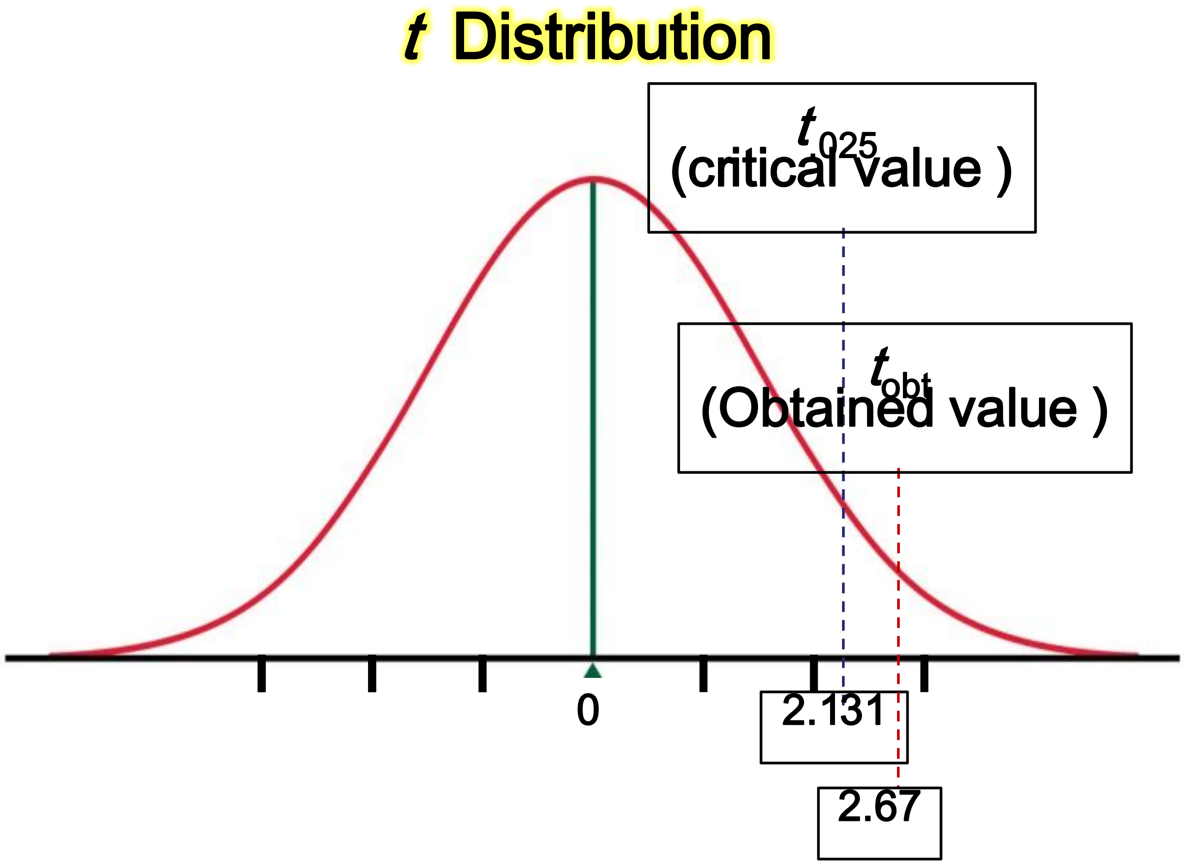
- The null hypothesis is rejected when:

$$t_{obt} \text{ is equal to or more extreme than } t_{.025}$$

- Where $t_{.025}$ is the critical value from the t distribution and is found using:

$$df = N - 1 \quad \text{OR} \quad 16 - 1 = 15$$

EXAMPLE 2: Test of the Null Hypothesis



- The null hypothesis is rejected since the obtained value is more extreme than the critical value (p = .05)

EXAMPLE 2: Statistical Conclusion and Interpretation

❶ "Since $t(15) = 2.67$, $p < .05$; Reject the null hypothesis.

❷ The observed difference between average MPG for the new model truck and average MPG for the previous model truck is not solely due to random error ($p = .05$). It appears that, on the average, the new model light truck gets slightly better gas mileage compared to the previous model ($p = .05$).

WHAT CAN GO WRONG WHEN WE REJECT OR FAIL TO REJECT THE NULL HYPOTHESIS?

❶ If the null hypothesis is really true:

- In the long-run (p = .05) we will reject the null hypothesis when we should have failed to reject it
- This is known as a TYPE I Error

❷ If the null hypothesis is really false:

- In the long-run (p = .05) we will fail to reject the null hypothesis when we should have rejected it
- This is known as a TYPE II Error

WHAT CAN GO WRONG WHEN WE REJECT OR FAIL TO REJECT THE NULL HYPOTHESIS?

❶ By reducing the *p* value from .05 to .01 we reduce the chance in the long-run of committing a Type I error, but we increase the chance of committing a Type II error

❷ By increasing the *p* value from .01 to .05 we reduce the chance in the long-run of committing a Type II error, but we increase the chance of committing a Type I error

HOW DO WE SOLVE THIS DILEMMA?

WHAT IS STATISTICAL POWER?

1. Statistical Power is the probability of detecting a real effect of the independent variable if a real effect exists

2. What affects statistical power?
   - Effect size
   - Sample size
   - Measurement error
   - *p* value (.05 versus .01)
   - Outliers

WHAT INCREASES STATISTICAL POWER?

1. Increasing the effect size of the IV
2. Increasing sample size
3. Reducing measurement error
4. Increasing the *p* value from .01 to .05

DECISION TABLE:

Statistical Conclusion	Reality: H_o is False	Reality: H_o is True
Reject H_o	Correct POWER	TYPE I Error
Fail to Reject H_o	TYPE II Error	Correct

WHAT IS PRACTICAL SIGNIFICANCE?

❶ Just because a result is significant doesn't mean it's applicable to everyday life (not practical)

❷ Very large sample size can result in statistical significance but not be practical to everyday life

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THAT'S IT FOR CHAPTER 8