<u>Writing the Equation of a Rational Function Given its Graph</u>

Note: VA = Vertical Asymptote HA = Horizontal Asymptote

1. Given: One VA = b, HA = 0, and a point (x,y): {plug in the value for "b" in the equation}

$$Use \rightarrow \boldsymbol{f(x)} = \frac{\boldsymbol{a}}{\boldsymbol{x-b}}$$

Use the given point (x,y) plugging in y for f(x) and x for x to solve for "a."

---

2. Given: One VA = c, HA = a, and one XI (b,0): {plug in the values for "c, a, & b" in the equation}

$$Use \rightarrow \boldsymbol{f(x)} = \frac{\boldsymbol{a(x-b)}}{\boldsymbol{x-c}}$$

The given point (x,y) is only to check your answer; you don't need it to solve the problem.

---

3. Given: Two VA = b & c, HA = 0, and a point, (x,y): {plug in the values for "b & c" in the equation}

$$Use \rightarrow \boldsymbol{f(x)} = \frac{\boldsymbol{a}}{\boldsymbol{(x-b)(x-c)}}$$

Use the given point (x,y) plugging in y for f(x) and x for x to solve for "a."

---

4. Given: Two VA = c & d, HA = 0, one XI (b,0) and a point (x,y) {plug in the values for "b, c, & d" in the equation}

$$Use \rightarrow \boldsymbol{f(x)} = \frac{\boldsymbol{a(x-b)}}{\boldsymbol{(x-c)(x-d)}}$$

Use the given point (x,y) plugging in y for f(x) and x for x to solve for "a."

---

5. Given: Two VA = d & e, HA = a, Two XI (b,0) & (c, 0) {plug in the values for "a, b, c, d, & e" in the equation}

$$Use \rightarrow \boldsymbol{f(x)} = \frac{\boldsymbol{a(x-b)(x-c)}}{\boldsymbol{(x-d)(x-e)}}$$

The given point (x,y) is only to check your answer; you don't need it to solve the problem.

---

6. Given: Two VA = d & e, HA = a, one XI (b,0) {plug in the values for "a, b, c (which is the same value as "b", d, & e" in the equation}

$$Use \rightarrow \boldsymbol{f(x)} = \frac{\boldsymbol{a(x-b)(x-c)}}{\boldsymbol{(x-d)(x-e)}}$$

The sneaky trick here is to use the same XI twice. That is, c=b.

The given point (x,y) is only to check your answer; you don't need it to solve the problem.

<u>Examples of Writing the Equation of a Rational Function Given its Graph</u>

1. Vertical asymptote x = –3, and horizontal asymptote y = 0. The graph has no x-intercept, and passes through the point (–2,3)

   a. $f(x) = \frac{3}{x+3}$ ← x– (–3) = x+ 3

   $3 = \frac{a}{(-2+3)} \rightarrow 3 = \frac{a}{1} \rightarrow 3 = a$

2. Vertical asymptote x = 4, and horizontal asymptote y = –2. The graph also has an x-intercept of 1, and passes through the point (–2,3)

   a. $f(x) = \frac{-2(x-1)}{(x-4)}$

3. Vertical asymptotes x = –1, x = –6, and horizontal asymptote y = 0. The graph has no x-intercept, and passes through the point (–4,–1)

   a. $f(x) = \frac{6}{(x+1)(x+6)}$

4. Vertical asymptotes x = –5, x = –1, and horizontal asymptote y = 0. The graph has an x-intercept of 1, and passes through the point (–2,2)

   a. $f(x) = \frac{2(x-1)}{(x+1)(x+5)}$

5. Vertical asymptotes x = –6, x = –2, and horizontal asymptote y = –3. The graph also has x-intercepts of –4 and 1, and passes through the point (0,1)

   a. $f(x) = \frac{-3(x+4)(x-1)}{(x+2)(x+6)}$

6. Vertical asymptotes x = –5, x = 2, and horizontal asymptote y = –3. The graph also has an x-intercept of 4 and, passes through the point (–4,2)

   a. $f(x) = \frac{-3(x-4)(x-4)}{(x+5)(x-2)}$

Here is another example of tricky type 6:

6. Vertical asymptotes x = –4, x = 1, and horizontal asymptote y = –4. The graph also has an x-intercept of –2 and, passes through the point (–3,1)

   a. $f(x) = \frac{-4(x+2)(x+2)}{(x-1)(x+4)}$